Conserved-mass aggregation model with mass-dependent fragmentation on complex networks

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We study the conserved-mass aggregation model with mass-dependent fragmentation on random networks (RNs) and scale-free networks (SFNs) with degree distribution $P(k) \sim k^{-\gamma}$. In the model, masses isotropically diffuse with unit rate. With rate ω, a mass m^{λ} is fragmented from a node with mass m, and moves to one of the linked nodes with the equal probability. For $\lambda=0$, the model is known to undergo the condensation phase transitions at a certain criticality ρ_c. From the mean-field balance equation for an aggregate, we analytically show that the present model exhibits different behavior depending on network structures. The condensation always occurs for $\lambda<0$. For $0<\lambda<1$, finite-sized systems on RNs and SFNs with $\gamma>3$ undergoes the condensation transitions (sharp crossovers) at ρ_c which diverges with network size N as N^{λ}. Hence, in the limit $N \to \infty$, masses uniformly distribute without the condensation (fluid phase). On the other hand, for $\gamma \leqslant 3$, a crossover $\lambda_c=1/(\gamma-1)$ exists. The condensation always occurs for $\lambda<\lambda_c$, while the fluid phase is always stable at any nonzero density for $\lambda \geqslant \lambda_c$. The phase separation results from the competition between the heterogeneity of network structure and the enhanced chipping by λ. We numerically confirm all the predictions.

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I. INTRODUCTION

Nonequilibrium condensation phenomena have been observed in a wide variety of transport systems ranging from traffic flow to polymer gels [1–8]. The nonequilibrium steady states of these systems are classified into two types of phases, the condensed phase and the fluid phase, respectively. In the condensed phase, a finite fraction of total particles condenses on a single site. On the other hand, the particle number of each site fluctuates around the total particle density ρ without the condensation. These systems evolve via basic microscopic dynamics ubiquitous in nature such as diffusion, aggregation upon contact, and fragmentation of aggregates. As the rates of these processes vary, a system may undergo the condensation transitions between the two phases at a certain critical density ρ_c [9–13].

The simplest mass-transport model exhibiting the condensation is zero-range process (ZRP) [9]. In ZRP, each site may contain an integer mass and unit mass can hop to one of the nearest-neighboring sites. The hopping rate depends on the mass of the departure site. When the nearest neighbor site is already occupied, mass aggregation takes place. The chipping (unit-mass dissociation) and aggregation processes of ZRP describe various condensations such as traffic jam [1], bunching of buses [2], coalescence of shaken steel balls [3]. On scale-free networks, recent studies showed that ZRP exhibits nontrivial dependence of the condensation on the network properties [10,11].

Another important class of condensation transitions emerges when the mass of a site is allowed to diffuse, in addition to chipping and aggregation. These processes arise in various phenomena such as polymer gels [4], the formation of colloidal suspensions [5], river networks [6,7], and cloud formation [8]. Conserved-mass aggregation (CA) model is the simplest one incorporating diffusion, chipping and aggregation upon contact [12–18]. In one-dimensional CA model, the mass m_i of a site i moves either to site $i-1$ or to site $i+1$ with unit rate, and then $m_i \to 0$ and $m_{i\pm1} \to m_{i\pm1} + m_i$. With rate ω, unit mass chips off from site i and moves to one of the nearest neighboring sites; $m_i \to m_i - 1$ and $m_{i\pm1} \to m_{i\pm1}+1$. The generalization to higher dimensions is straightforward. As total masses are conserved, the density ρ and the rate ω determine the phase of CA model. The condensation transition arises from the competition between diffusion and chipping process.

In the symmetric CA (SCA) model [12,13] in which diffusion and chipping direction are unbiased, the condensation transitions take place at a certain ρ_c. In SCA model, the single site mass distribution $P(m)$ was shown to undergo phase transitions on regular lattices [12]. For a fixed ω, as ρ varies across the critical value $\rho_c(\omega)$, the behavior of $P(m)$ for large m was found to be [12]

$$P(m) \sim \begin{cases} e^{-m/m^*}, & \rho < \rho_c(\omega), \\ m^{-\tau}, & \rho = \rho_c(\omega), \\ m^{-\tau} + (\text{infinite aggregate}), & \rho > \rho_c(\omega), \end{cases} \tag{1}$$

where $\rho_c(\omega)=\sqrt{\omega+1}-1$ and $\tau=5/2$. ρ_c and τ are independent of the spatial dimension d [13]. The steady state $P(m)$ is not factorized and not known exactly. Nevertheless, a mean field approach, which assumes a factorized steady state, does in fact give the correct phase diagram in the case of symmetric dynamics. Recent studies showed that the existence of the condensation transitions depends on the spatial disorder [14], the symmetry of moving directions [15], the constraints on the rate of diffusion [16], and also the underlying network structure [17].

In more general situations, the rate of diffusion and of chipping will depend on the mass of a departure site [12]. When the diffusion rate depends on mass as $D(m) \sim m^{-\alpha}$ with $\alpha>0$ [16], the condensed phase disappeared in the ther-

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modynamic limit. On the other hand, the mass-dependent fragmentation is also important in general physical situation. For instance, in gelation phenomena, there is no reason why only one monomer at the end of a polymer is chipped regardless of the mass of polymers. It is very likely that more monomers can be chipped from more massive polymers. Hence it is natural to study the effect of mass-dependent fragmentation on the condensation phase transitions.

Recently, we studied SCA model with mass-dependent fragmentation in one dimension [18]. In the model, the fragmented mass δm_i from a site i is replaced with $\delta m_i = m_i^\lambda$ with $\lambda < 1$. Evolution rules are the same as those of the ordinary SCA model. We found that condensation transitions take place only in finite-sized systems. However, the ρ_c diverges with the system size N as N^β. Hence in the limit $N \to \infty$, no condensation transitions occur so a system is always in the fluid phase for $0 < \lambda < 1$. Since the transition only occurs in the finite systems, it might be called the sharp crossover from a condensed phase to a fluid phase instead of transitions. However in the finite systems, the transitions exhibit the same type of the transitions as those of SCA model [12] with the different critical exponents and diverging critical density. Hence we use the term "transition" which seems more suitable to characterize the related phenomena. It was argued from a mean-field-type theory that there are no condensation transitions in any dimensions.

In this paper, as the generalization of the previous study [18], we investigate the condensation phenomena of SCA model with mass-dependent fragmentation on complex scale-free networks (SFNs) with degree distribution $P(k) \sim k^{-\gamma}$. From a mean-field-type balance equation for the mass of an aggregate, we examine the stability of an aggregate against λ. On random networks (RNs) and SFNs with $\gamma > 3$, the condensation always occurs for $\lambda < 0$. However, no condensations take place for $\lambda > 0$. Only for $\lambda = 0$, the condensation transitions takes place at finite ρ_c [12]. We also find that the exponent β should be λ, i.e., $\beta = \lambda$, which gives the exponent τ in terms of λ, i.e., $\tau = (\lambda + 2)/(\lambda + 1)$. The mean-field predictions agree well with the results in one dimension [18]. We also numerically confirm the mean-field predictions.

On the other hand, for $\gamma \leqslant 3$, a system exhibits the quite different behavior from the mean-field one for $\gamma > 3$. For $\gamma \leqslant 3$, the phase separation takes place at a certain λ_c as λ varies. For $\lambda < \lambda_c$, the condensation always occurs. However, for $\lambda \geqslant \lambda_c$, a system is always in the fluid phase without the condensation. From the mean-field argument, we obtain $\lambda_c = 1/(\gamma - 1)$ and numerically confirm the relation.

The outline of this paper is as follows. We introduce the SCA model with mass-dependent fragmentation on complex networks in Sec. II. The condensation phenomena on RNs and SFNs are discussed in Secs. III and IV. Finally, we summarize our results in Sec. V.

II. SCA MODEL WITH MASS-DEPENDENT FRAGMENTATION ON COMPLEX NETWORKS

We consider a network with N nodes and K links. The degree k_i of a node i is defined as the number of links connected to other nodes. The average degree of a node $\langle k \rangle$ is given as $\langle k \rangle = 2K/N$. The degree distribution $P(k)$ is a Poisson distribution for RNs and a power-law distribution of $P(k) \sim k^{-\gamma}$ for SFNs. For the construction of SFN, we use a static model [19] instead of preferential attachment algorithm [20]. N nodes are indexed by an integer i $(i = 1, \ldots, N)$. The weight $p_i = i^{-\alpha}$ is assigned to each node, where α is a control parameter in [0,1). Next select two different nodes i and j with probabilities $p_i / \Sigma_1^N p_k$ and $p_j / \Sigma_1^N p_k$, and add a link between them unless a link already exists. We repeat this process until the number of total links is K. The degree exponent γ is given as $\gamma = (1 + \alpha)/\alpha$ [19]. In static model, it is desired to use large $\langle k \rangle$ to construct fully connected networks. In simulations, we use $\langle k \rangle = 4$.

Each node may have an integer number of particles, and the mass on a node is defined as the number of particles on the node. Initially M particles are randomly distributed over N nodes with a given conserved density $\rho = M/N$. Next a node i with mass m_i is randomly chosen. A node j among the nodes directly linked to i is also chosen randomly. Then one of the following events is taken.

(i) Diffusion: With unit rate, the mass m_i moves to the node j. If the node j has already mass m_j, then the aggregation takes place; $m_i \to 0$ and $m_j \to m_j + m_i$.

(ii) Chipping: With rate ω, mass $\delta m_i = A m_i^\lambda$ is fragmented from m_i and moves to the node j; $m_i \to m_i - \delta m_i$ and $m_j \to m_j + \delta m_i$.

The $\lambda = 0$ and $A = 1$ case is the ordinary SCA model [12]. The $\lambda = 1$ case is a trivial point, but two extreme situations emerge according to the values of A. For $A = 0$ or 1, masses only diffuse without chipping so that all masses aggregate on a single node (complete condensation). On the other hand, for $0 < A < 1$, the fragmented mass is proportional to the whole mass, which means that an infinite aggregates cannot exist. Hence, the system is always in the fluid phase for the $\lambda = 1$ and $0 < A < 1$ case, which is physically very similar to the random fragmentation model [21]. In this paper, we consider the model with $\lambda < 1$ and $A = 1$ on RNs and SFNs.

III. THE CONDENSATION PHENOMENA ON RNS AND SFNS WITH $\gamma > 3$

The CA model is well described by mean-field theory in any dimensional regular lattice [12–16] and on SFNs [17]. Hence we expect that mean-field theory also describes the present model on RNs and SFNs. From the mean-field rate equation for an aggregate, we examine the stability of the condensed phase against λ in the steady state and obtain the exponent β.

Let us assume that a system is in the condensed phase. We assume that the background masses are uniformly distributed over the nodes with the density ρ_c for the mean-field-type approximation. In addition, an aggregate with mass m_a exists on a node i. The time dependence of $m_a(t)$ is obtained by considering the gain and the loss of m_a from diffusion and chipping processes. First, the gain from the diffusion process is written as

$$\sum_{j=1}^{k_i}\sum_{m=0}^{\infty}(H_{ji}+H_{ij})mP_j(m,t). \quad (2)$$

H_{ij} is the hopping probability from the node j to i, and $P_j(m,t)$ is the mass distribution of node j at time t. k_i is the degree of a node i. Similarly, the gain and the loss from chipping process is

$$\omega\sum_{j=1}^{k_i}\sum_{m=0}^{\infty}m^{\lambda}H_{ij}P_j(m,t)-\omega m_a^{\lambda}. \quad (3)$$

For the symmetric CA model, one of the linked nodes is randomly selected for the diffusion or chipping and thus $H_{ij}=1/k_j$. Since the degree fluctuation is finite on RNs and SFNs with $\gamma>3$, we approximate $k_i=\langle k\rangle$ for all i for the mean-field approximation. Then H_{ij} is $H_{ij}=1/\langle k\rangle$ and $\Sigma_j(H_{ij}+H_{ji})=2$. We also approximate $P_j(m,t)=P(m,t)$. Then combining Eqs. (2) and (3), one obtains the equation for m_a in the steady state as

$$dm_a(t)/dt=2\langle m\rangle+\omega\langle m^{\lambda}\rangle-\omega m_a^{\lambda}=0, \quad (4)$$

where $\langle m^{\lambda}\rangle=\Sigma_{m=0}^{\infty}m^{\lambda}P(m)$ and $\langle m\rangle=\Sigma_{m=0}^{\infty}mP(m)$ is an average mass of a node. $P(m)$ is the mass distribution of a node in the steady state. In the condensed phase, the mass of a node fluctuates around a critical density ρ_c and $\langle m\rangle=\rho_c$. Neglecting the density fluctuation, the average $\langle m^{\lambda}\rangle$ is approximately given as $\langle m^{\lambda}\rangle\sim\Sigma_m m^{\lambda}P(m)\delta(m-\rho_c)\sim\rho_c^{\lambda}P(\rho_c)$. The occupation probability s of a node having nonzero mass satisfies $s=\Sigma_{m>0}P(m)$ which is approximately equal to $s_c=P(\rho_c)$ for $\rho\geqslant\rho_c$. The s_c is s at $\rho=\rho_c$. With $\langle m\rangle=\rho_c$ and $\langle m^{\lambda}\rangle\sim\rho_c^{\lambda}s_c$, Eq. (4) becomes

$$2\rho_c+\omega\rho_c^{\lambda}s_c=\omega m_a^{\lambda}. \quad (5)$$

To examine the stability of the condensed phase, we first consider the $\lambda=0$ case. With $\lambda=0$, we have $\rho_c=\omega(1-s_c)/2$, which coincides with the result of Ref. [12]. Since the finite s_c gives a finite $\rho_c(<\infty)$, the condensed phase is stable. Hence the condensation transitions take place at finite ρ_c [12]. For $\lambda<0$, m_a scales with network size N as $m_a\sim N$. The right-hand side (RHS) of Eq. (5) thus goes to zero in the limit $N\to\infty$. Therefore we have a trivial solution, $\rho_c=s_c=0$ for $\lambda<0$ in the limit $N\to\infty$, which means that the complete condensation without the background distribution always occurs for any nonzero ρ. For $\lambda<0$, the time needed to dissipate an aggregate exponentially increases with the mass of an aggregate. As a result, once a massive aggregate is formed, it is always stable against the chipping process.

Finally, for $0<\lambda<1$, the RHS of Eq. (5) diverges with N and we have $\rho_c=\infty$ in the limit $N\to\infty$. Hence the condensed phase is unstable, and eventually disappears in the limit $N\to\infty$. For finite-sized systems, $P(m)$ scales as $P(m)=m^{-\tau}f(m/N^{\phi})$ at $\rho=\rho_c$. The exponent ϕ is the crossover exponent [13]. From $\rho_c=\int_1^{\infty}mP(m)dm$, ρ_c scales with N as N^{β} with $\beta=\phi(2-\tau)$. Due to $\tau<2$ for $0<\lambda<1$ [18], ρ_c diverges as $\rho_c\sim N^{\beta}$. Hence, from Eq. (5), we find $N^{\beta}\sim N^{\lambda}$ up to the first order of the LHS, which gives $\beta=\lambda$.

From the normalization condition of $P(m)$, one obtains another scaling relation $\phi(\tau-1)=1$ [13]. Together with $\beta=\phi(2-\tau)$, one obtain the scaling relation $\tau=(\beta+2)/(\beta+1)$ [18]. With $\beta=\lambda$, we get $\tau=(\lambda+2)/(\lambda+1)$. In summary, we obtain the exponents, β and τ for $0<\lambda<1$ as

$$\beta=\lambda, \quad \tau=(\lambda+2)/(\lambda+1). \quad (6)$$

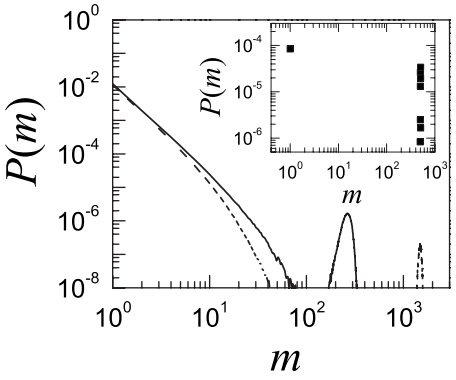

FIG. 1. The plot of $P(m)$ for $\lambda<0$ on RNs with $\rho=0.05$ and $\omega=0.1$. The main plot shows $P(m)$ of $\lambda=-0.1$ with $N=10^4$ (solid line) and 5×10^4 (dashed line), respectively. The inset shows $P(m)$ of $\lambda=-1.0$ with $N=10^4$.

To check the mean field predictions, we perform Monte Carlo simulations on RNs and SFNs with $\gamma>3$. With the random initial distributions of masses, the system is allowed to reach the steady states. We run simulations typically up to $t=10^7$ time steps for size N up to 10^4. In the steady states, we measure $P(m)$ for various values of λ. We perform simulations for various ω, but we find no essential differences. Hence, in what follows, we set $\omega=0.1$.

We use the following algorithm for determining the mass δm chipped from a node i with mass m_i. For $\lambda>0$, the mass $\delta m(=m_i^{\lambda})$ is not an integer. In that case, integer $[m_i^{\lambda}]$ masses are chipped and then the unit mass is chipped with the probability $m_i^{\lambda}-[m_i^{\lambda}]$. The symbol $[x]$ denotes an integer not greater than the real number x. For $\lambda\leqslant0$, only the unit mass is chipped with the probability m_i^{λ}.

For $\lambda<0$, we perform simulations on RNs with $\rho=0.05$ and N up to 5×10^4. Figure 1 shows $P(m)$ for $\lambda=-0.1$. As shown, an aggregate always exists with an exponential background distribution even for very low density. Since the background distribution decreases with N, all masses condense on a single node in the limit $N\to\infty$ as expected. For $\lambda=-1.0$, the complete condensation is clearly shown (inset). Hence, we believe that for $\lambda<0$, the complete condensation always occurs for any ρ, which agrees well with the mean-field prediction.

For $0<\lambda<1$, finite-sized systems undergos the condensation phase transitions as shown in Fig. 2. However, the condensed phase eventually disappears in the limit $N\to\infty$ and ρ_c diverges with N. To find the size dependence of ρ_c, we measure ρ_c for each N as follows [18]. In the steady state, $P(m)$ scales as [13]

$$P(m)=m^{-\tau}f(m/N^{\phi})+\frac{1}{N}\delta[m-(\rho-\rho_c)N]. \quad (7)$$

From the conservation of total masses, ρ_c is given as $\rho_c=\rho-\rho_{\infty}$, where ρ_{∞} is the density of an aggregate. From the fact

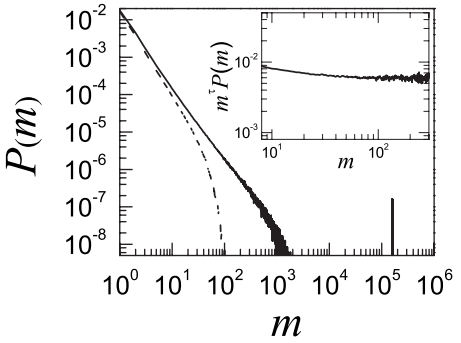

FIG. 2. The plot of $P(m)$ for $\lambda=0.1$ on RNs with $N=8\times10^3$. In the condensed phase of $\rho=20$ (solid line), $P(m)$ algebraically decays with $\tau=1.8(1)$ (<2). In the fluid phase with $\rho=0.1$ (dashed line), $P(m)$ exponentially decays without any aggregates. Inset shows the scaling plot $m^{\tau}P(m)$ with $\tau=1.8$. We use $P(m)$ of $\rho=20$ for the scaling plot.

that the background distribution does not change for $\rho \geqslant \rho_c$, one can estimate ρ_c from the relation $\rho_c=\int_1^{m_0} mP(m)dm$ in the condensed phase, where m_0 is the cutoff mass at which the background distribution terminates.

We measure $P(m)$ for $\lambda=0.1$, 0.25, and 0.35 in the condensed phase with sufficiently high density. We set $\rho=20$ for each λ on RNs. On SFNs with $\gamma=4.3$, we set $\rho=3$ for $\lambda=0.1$ and 0.25, and $\rho=10$ for $\lambda=0.35$. Using the above method, we estimate ρ_c. In Fig. 3, we plot ρ_c as a function of N on RNs and SFNs with $\gamma=4.3$. Insets show the plots of β against λ. We estimate β using the local slope β_{eff} defined as $\beta_{\text{eff}}=\ln[\rho(N)/\rho(N/b)]/\ln b$ with $b=2$ for each λ (not shown). Since we are interested in the value of the exponent β in the thermodynamic limit $N\to\infty$, we should examine how the local slope changes as N increases. As shown, ρ_c diverges as N^{β} with $\beta=\lambda$ for both RNs and SFNs with $\gamma=4.3$, which agree well with the mean field predictions. To check the scaling relation $\tau=(\lambda+2)/(\lambda+1)$, we measure τ on RN with $\lambda=0.1$ (Fig. 2). Using the scaling plot $m^{\tau}P(m)$ (inset of Fig. 2), we estimate $\tau=1.8(1)$. We also estimate $\tau=1.85(5)$ for SFN of $\gamma=4.3$ with $\lambda=0.1$, $\rho=3$ and $N=8\times10^3$ (not shown). The estimates of τ agree well with the prediction $\tau=1.91$ for $\lambda=0.1$.

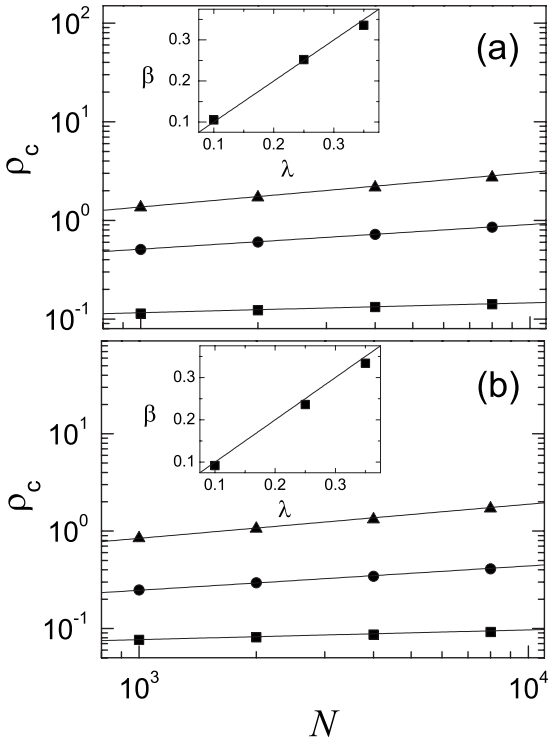

FIG. 3. The double logarithmic plot of $\rho_c(N)$ on RNs (a) and SFNs with $\gamma=4.3$ (b). In each panel, symbols from top to bottom correspond to $\lambda=0.35$, 0.25, and 0.1, respectively. Each line is the guide to the eye with the slope λ. The insets shows the plot of β against λ. The line in each inset is $\beta=\lambda$.

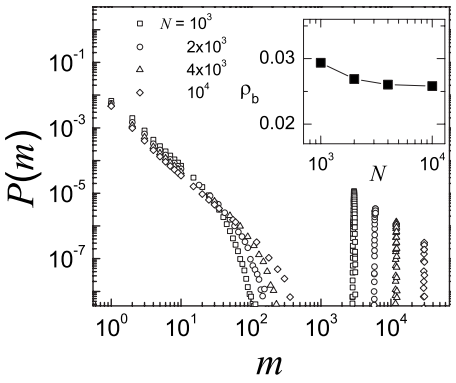

FIG. 4. $P(m)$ for $\lambda=0.1$ on SFNs with $\gamma=2.7$. The inset shows ρ_b against size N.

IV. THE CONDENSATION PHENOMENA ON SFNS WITH $2\leqslant\gamma\leqslant3$

On SFNs with $2\leqslant\gamma\leqslant3$, $\langle k^2\rangle$ diverges, and so we cannot neglect the degree fluctuation. It means that $P_j(m)$ and H_{ij} strongly depend on the degree of the node j, so that the mean-field approximations for Eq. (5) are not valid for $\gamma\leqslant3$. In the ordinary SCA model on SFNs with $2\leqslant\gamma\leqslant3$ [17], an aggregate always forms at the hub node with the maximal degree $k_{\max}$ due to the hub structure of the networks. The aggregate diffuses around a network and causes a background distribution which is neither simple power law nor exponential. We call the background distribution quasiexponential because it decays exponentially only for sufficiently large m due to the finite size effect [17]. Hence, if an aggregate exists, then it forms at the hub node in the present model as well. The background distribution $P(m)$ is also quasiexponential in the present model. Figure 4 shows the $P(m)$ of $\gamma=2.7$, $\lambda=0.1$ and $\omega=0.1$ for various size N up to 10^4. The cutoff mass m_0 increases with N, which indicates that the background distribution is stable in the thermodynamic limit $N\to\infty$.

The background mass density ρ_b is given as $\rho_b=\Sigma^{m_0} mP(m)$, where m_0 is the cutoff mass. Since ρ_b saturates to a small value as shown in the inset of Fig. 4, the mass gain of an aggregate from background masses is negligible compared with the fragmented mass δm_a from an aggregate of mass m_a, $\delta m_a=m_a^{\lambda}\sim N^{\lambda}$ with $\lambda\geqslant0$. Hence, we neglect the mass change of an aggregate by the background distribution.

To simplify calculation, we further assume the following situations. First, we assume that an aggregate stays at the hub node due to the strong hub structure for $\gamma \leqslant 3$. This assumption is based on the fact that since the probability of finding a walker on the hub node is proportional to $k_{\max}$ [22], an aggregate, if exists, spends most time on the hub node. Secondly, we neglect the diffusion of the chipped mass δm_a from an aggregate on the hub node. Two walkers starting at the same node meet again within finite time interval for $\gamma \leqslant 3$ [17], which means that δm_a returns to the hub within finite time interval. Hence, we neglect the mass change of an aggregate by the diffusion of the chipped mass δm_a. Therefore, neglecting the diffusion of masses, we consider the the gain and the loss of an aggregate by the chipping process to examine the stability of an aggregate against λ for $2 \leqslant \gamma \leqslant 3$.

Based on the above assumptions, we only consider an aggregate with mass m_a on the hub node with the degree $k_{\max}$ without background masses for simplicity. Then, the loss of m_a from the chipping process is $m_a^\lambda \sim N^\lambda$. After the chipping, the fragmented mass m_a^λ moves to one of the linked nodes with probability $1/k_{\max}$. Hence, each node linked to the hub node has $m_a^\lambda / k_{\max}$ in average. The gain from the linked nodes by the chipping process is

$$\sum_{l=1}^{k_{\max}} (m_a^\lambda / k_{\max})^\lambda / k_l, \tag{8}$$

where k_l^{-1} is the hopping probability from node l to the hub node. We approximate the sum $\Sigma_l 1/k_l$ to the average $\langle \Sigma_l 1/k_l \rangle_{\text{hub}}$. $\langle \cdots \rangle_{\text{hub}}$ denotes the average over the nodes linked to the hub. Then we have $\langle \Sigma_l^{k_{\max}} 1/k_l \rangle_{\text{hub}} = k_{\max} \int^{k_{\max}} g(k)/k \, dk$, where $g(k)$ is the degree distribution in the sum. In the limit $N \to \infty$, the number of terms in the sum $(k_{\max})$ diverges, so that we approximate $g(k)$ to $P(k)$. Then we have $\langle \Sigma_l 1/k_l \rangle_{\text{hub}} \sim k_{\max} \langle 1/k \rangle$ with $\langle 1/k \rangle = \int^{k_{\max}} P(k)/k \, dk$. Since $\langle 1/k \rangle$ is finite, we have $\langle \Sigma_l 1/k_l \rangle_{\text{hub}} \sim k_{\max}$. Hence, we approximately obtain the gain (8) as $m_a^{\lambda^2} k_{\max}^{1-\lambda}$. The condition for the stable aggregate is $m_a^{\lambda^2} k_{\max}^{1-\lambda} > m_a^\lambda$. With $m_a \sim N$ and $k_{\max} \sim N^{1/(\gamma-1)}$ [20], one gets

$$N^\lambda < N^{\lambda^2 + (1-\lambda)/(\gamma-1)} \tag{9}$$

for the stable aggregate. From Eq. (9), the crossover λ_c is given as

$$\lambda_c = 1/(\gamma - 1). \tag{10}$$

For $\lambda < \lambda_c$, an aggregate is stable in the limit $N \to \infty$, and the condensation always occurs as in the ordinary SCA model on SFNs with $2 \leqslant \gamma \leqslant 3$ [17]. On the other hand, for $\lambda \geqslant \lambda_c$, the aggregate eventually disappears by repeated chipping processes and the fluid phase is stable in the limit $N \to \infty$. As a result, a system undergoes the phase separation at λ_c. The phase separation results from the competition between the heterogeneous network structure and the enhanced chipping by λ. The result (10) itself of our model is the same as that of ZRP on SFNs [10], even though the details of our model are different from those of ZRP. For $\lambda < \lambda_c$, the background distribution emerges by the diffusion of masses in the present model, while the complete condensation occurs in ZRP.

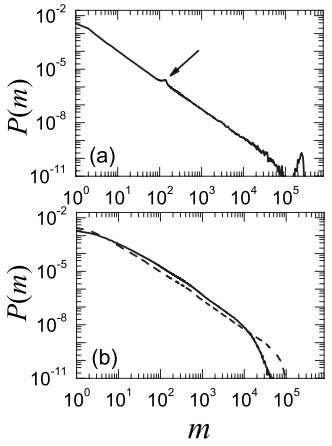

FIG. 5. The plot of $P(m)$ on SFNs with $\gamma=2.7$. With $\omega=0.1$, $\rho=3.0$, and $N=10^5$, (a) $\lambda=0.4$ and (b) $\lambda=0.588$ (dashed line) and 0.7 (solid line). The arrow in (a) indicates the second peak.

We perform simulations on SFNs with $\gamma=2.7$ to confirm the mean-field prediction, i.e., the existence of λ_c. From Eq. (10), one reads $\lambda_c \approx 0.588$ for $\gamma=2.7$. We run simulations for $\lambda=0.4$, 0.588, and 0.7 up to 10^7 time steps for $N=10^5$. We set $\omega=0.1$. In Fig. 5, we plot $P(m)$ in the steady state. For $\lambda=0.4(<\lambda_c)$, an aggregate forms with the mass in the order of N [Fig. 5(a)]. At $\lambda_c=0.588$ and $\lambda=0.7$, $P(m)$ decays without the condensation [Fig. 5(b)]. These numerical results agree well with the mean-field predictions.

On the other hand, for $\lambda=0.4$ [Fig. 5(a)], $P(m)$ exhibits another peak at small mass in addition to the peak of an infinite aggregate. In the condensed phase, the fragmented mass δm from an infinite aggregate with mass m_a is $\delta m = m_a^\lambda$. In the steady state, the aggregate with mass δm is also stable. As a result, $P(m)$ exhibits another peak at δm which is the order of N^λ. For instance, m_a for $\lambda=0.4$ is about $m_a=3 \times 10^5$. As shown in Fig. 5(a), another peak is at about $m_{2\text{nd}} = m_a^\lambda = 155$ as expected. The second peak causes the third peak at about $m_a^{\lambda^2}$ and so on. In this way, one may find several peaks for sufficiently large λ in the condensed phase.

V. SUMMARY

In summary, we investigate the condensation phenomena of SCA model with mass-dependent fragmentation on complex networks. In the model, with rate ω, the mass $\delta m = m^\lambda$ is fragmented from a node with mass m and moves to one of the linked nodes. With unit rate, the diffusion process occurs. The $\lambda=0$ case is the ordinary SCA model [12]. From the balance equation for an aggregate, we examine the stability of an aggregate against λ. The present SCA model exhibits quite different behavior according to the network structure.

On RNs and SFNs with $\gamma > 3$, the complete condensation without background distribution always occurs for $\lambda < 0$. It comes from the fact that the time needed to dissipate an aggregate by repeated chipping processes exponentially diverges with the mass of an aggregate. On the other hand, for $0 < \lambda < 1$, only finite systems undergoes the condensation

transition at a certain critical density ρ_c, which diverges with networks size N as $\rho_c \sim N^{\lambda}$. Therefore, a system is always in the fluid phase without the condensation for any density ρ in the limit $N \to \infty$.

On SFNs with $2 \leqslant \gamma \leqslant 3$, the present model exhibits quite different behavior from that of the ordinary SCA model. There is a crossover λ_c given as $\lambda_c = 1/(\gamma - 1)$. The condensation with a quasiexponential background distribution always occurs at any density for $\lambda < \lambda_c$, while the fluid phase without the condensation is always stable at any density for $\lambda \geqslant \lambda_c$. As a result, a system undergoes the phase separation at λ_c as λ varies. The phase separation results from the competition between the heterogeneous network structure and the enhanced chipping by λ.

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